Théorie de la synchronisation - Equation de Duffing

Etude du régime transitoire - Oscillateur faiblement en avance sur l'excitation

Caractéristiques du système

$T := 0.2 \cdot s \qquad \omega_0 := \frac{2\cdot\pi}{T} \qquad J := 8\cdot 10^{-7}\cdot kg\cdot m^2 \qquad q_0 := 270\cdot deg$

Frottement visqueux

$\eta := 0.002 \qquad C := 2\cdot J\cdot\eta\cdot\omega_0 \qquad F_{v\_max} := C\cdot\omega_0\cdot q_0 \qquad \lambda := \frac{F_{v\_max}}{J\cdot\omega_0^{\,2}} \qquad h := 2\cdot\frac{\eta}{\lambda}$

$F_{v\_max} = 0.015\,N\cdot mm \qquad \lambda = 0.019 \qquad h = 0.212$

Frottement quadratique

$B := 0.05\cdot F_{v\_max} \qquad \beta_1 := \frac{B}{\lambda\cdot J\cdot\omega_0^{\,2}} \qquad \beta_1 = 0.05 \qquad F_{q\_max} := B\cdot q_0^{\,3} \qquad F_{q\_max} = 0.078\,N\cdot mm$

Régimes transitoires vers un foyer attractif

Excitation harmonique

$A_c := \frac{8\cdot h^3}{3\sqrt{3}} \qquad A_c = 0.015 \qquad \boxed{A := 2\cdot A_c} \qquad A = 0.029$

$a_1 := \sqrt{\frac{4\cdot A}{3\cdot\beta_1}} \qquad F_{harm} := a_1\cdot\left(\lambda\cdot J\cdot\omega_0^{\,2}\right) \qquad F_{harm} = 1.318\times 10^{-5}\,N\cdot m$

$n := 500 \qquad i := 0..n \qquad x_0 := 0 \qquad x_1 := 2\cdot\pi \qquad \Delta x := \frac{x_1 - x_0}{n} \qquad x_i := x_0 + i\cdot\Delta x$

$Y_i := \frac{a_1}{h}\cdot sin(x_i)\cdot(0 < x_i < \pi) \qquad \varepsilon := h \qquad \sqrt{3} = 1.732 \qquad \boxed{\varepsilon = 0.212}$

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X := | for i ∈ 0..n
     |   | Z ← polyracines((−a1·cos(xi)  ε  0  (−3/4)·β1)^T)
     |   | for j ∈ 0..2
     |   |   X(i,j) ← Zj·(Im(Zj) = 0)·(Re(Zj) > 0)
     | X
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$v := \begin{pmatrix} -A & \varepsilon^2 + h^2 & -2\cdot\varepsilon & 1 \end{pmatrix}^T \qquad Z := polyracines(v) \qquad y_s := \sqrt{\frac{4\cdot Z}{3\cdot\beta_1}} \qquad x_s := arccos\left[\frac{1}{a_1}\cdot\left(\varepsilon\cdot y_s - \frac{3}{4}\cdot\beta_1\cdot y_s^{\,3}\right)\right]$

$\frac{Z}{h} = \begin{pmatrix} 0.086 + 1.295i \\ 0.086 - 1.295i \\ 1.828 \end{pmatrix} \qquad y_s = \begin{pmatrix} 1.979 + 1.852i \\ 1.979 - 1.852i \\ 3.216 \end{pmatrix} \qquad x_s = \begin{pmatrix} 0.44 + 0.472i \\ 0.44 - 0.472i \\ 2.262 \end{pmatrix}$

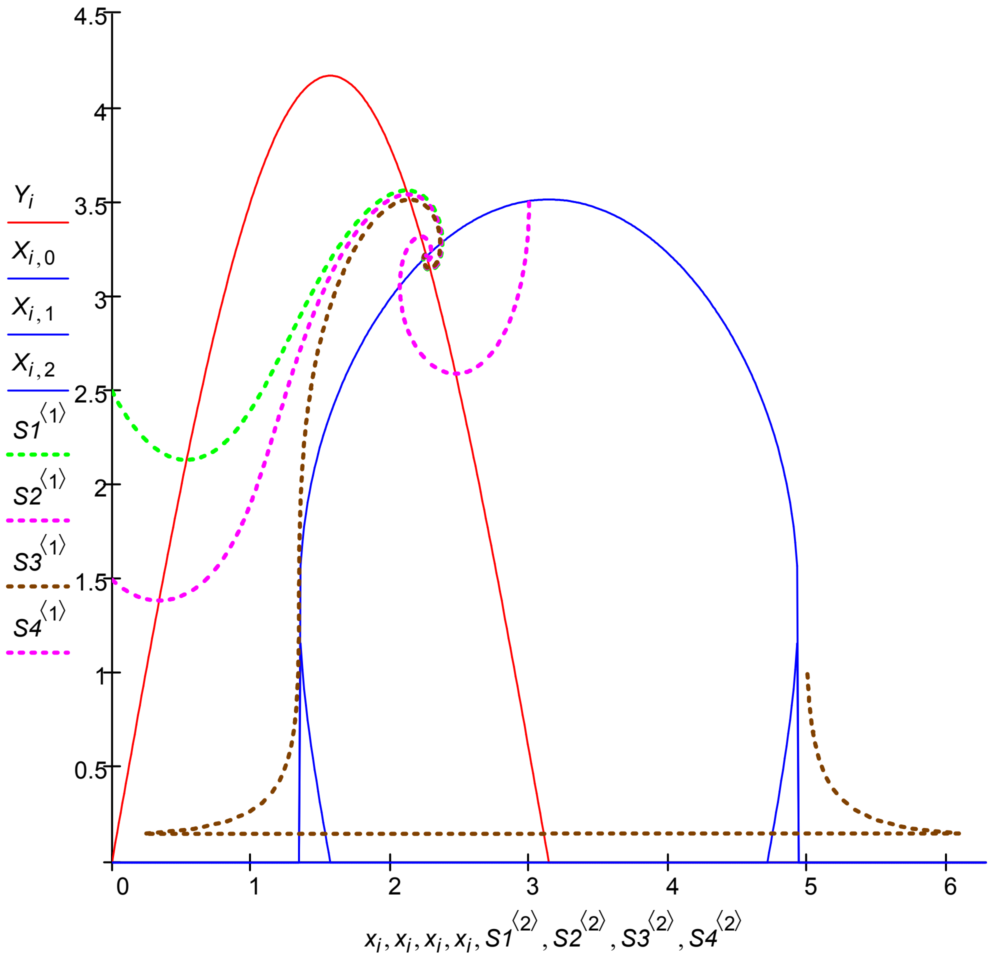
$Y0 := \begin{pmatrix} 2.5 \\ 0 \end{pmatrix} \qquad \mathbf{D}(t, Y) := \begin{bmatrix} \left(\frac{-h}{2}\cdot Y_0 + \frac{a_1}{2}\cdot sin(Y_1)\right)\lambda \\ \lambda\cdot\left[\frac{-\varepsilon}{2} + \frac{3}{8}\cdot\beta_1\cdot(Y_0)^2 + \frac{a_1}{2\cdot Y_0}\cdot cos(Y_1)\right] \end{bmatrix}$

$t_f := 4000$ $\qquad S1 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\qquad S1^{\langle 2\rangle} := mod(S1^{\langle 2\rangle}, 2\cdot\pi)$

$Y0 := \begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$ $\qquad S2 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\qquad S2^{\langle 2\rangle} := mod(S2^{\langle 2\rangle}, 2\cdot\pi)$

$Y0 := \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ $\qquad S3 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\qquad S3^{\langle 2\rangle} := mod(S3^{\langle 2\rangle}, 2\cdot\pi)$

$Y0 := \begin{pmatrix} 3.5 \\ 3 \end{pmatrix}$ $\qquad S4 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\qquad S4^{\langle 2\rangle} := mod(S4^{\langle 2\rangle}, 2\cdot\pi)$



Régimes transitoires vers un noeud attractif

Excitation harmonique

$\boxed{A := 0.4\cdot A_c}$ $\qquad A = 5.885\times 10^{-3}$ $\qquad \varepsilon := h$ $\qquad \boxed{\varepsilon = 0.212}$

$a_1 := \sqrt{\dfrac{4\cdot A}{3\cdot\beta_1}}$ $\qquad F_{harm} := a_1\cdot\left(\lambda\cdot J\cdot\omega_0^{\,2}\right)$ $\qquad F_{harm} = 5.896\times 10^{-6}\,N\cdot m$ $\qquad \sqrt{3} = 1.732$

$Y_i := \dfrac{a_1}{h}\cdot sin(x_i)\cdot(0 < x_i < \pi)$

$$X := \left|\begin{array}{l} \text{for } i \in 0..n \\ \quad \left|\begin{array}{l} Z \leftarrow polyracines\left(\left(-a_1\cdot cos(x_i) \quad \varepsilon \quad 0 \quad \dfrac{-3}{4}\cdot\beta_1\right)^T\right) \\ \text{for } j \in 0..2 \\ \quad X_{i,j} \leftarrow Z_j\cdot(Im(Z_j) = 0)\cdot(Re(Z_j) > 0) \end{array}\right. \\ X \end{array}\right.$$

$$v := \begin{pmatrix} -A & \varepsilon^2 + h^2 & -2\cdot\varepsilon & 1 \end{pmatrix}^T \qquad Z := polyracines(v) \qquad y_s := \sqrt{\frac{4\cdot Z}{3\cdot\beta_1}} \qquad x_s := arccos\left[\frac{1}{a_1}\cdot\left(\varepsilon\cdot y_s - \frac{3}{4}\cdot\beta_1\cdot {y_s}^3\right)\right]$$

$$\frac{Z}{h} = \begin{pmatrix} 0.488 \\ 0.756 + 0.831i \\ 0.756 - 0.831i \end{pmatrix} \qquad y_s = \begin{pmatrix} 1.662 \\ 2.306 + 1.02i \\ 2.306 - 1.02i \end{pmatrix} \qquad x_s = \begin{pmatrix} 1.098 \\ 1.022 + 0.914i \\ 1.022 - 0.914i \end{pmatrix}$$

$$Y0 := \begin{pmatrix} 2.5 \\ 0 \end{pmatrix} \qquad \boldsymbol{D}(t, Y) := \begin{bmatrix} \left(\frac{-h}{2}\cdot Y_0 + \frac{a_1}{2}\cdot sin(Y_1)\right)\lambda \\ \lambda\cdot\left[\frac{-\varepsilon}{2} + \frac{3}{8}\cdot\beta_1\cdot(Y_0)^2 + \frac{a_1}{2\cdot Y_0}\cdot cos(Y_1)\right] \end{bmatrix}$$

$$t_f := 4000 \qquad S1 := rkfixe(Y0, 0, t_f, n, \boldsymbol{D}) \qquad S1^{\langle 2\rangle} := mod\left(S1^{\langle 2\rangle}, 2\cdot\pi\right)$$

$$Y0 := \begin{pmatrix} 1.5 \\ 0 \end{pmatrix} \qquad S2 := rkfixe(Y0, 0, t_f, n, \boldsymbol{D}) \qquad S2^{\langle 2\rangle} := mod\left(S2^{\langle 2\rangle}, 2\cdot\pi\right)$$

$$Y0 := \begin{pmatrix} 1 \\ 5 \end{pmatrix} \qquad S3 := rkfixe(Y0, 0, t_f, n, \boldsymbol{D}) \qquad S3^{\langle 2\rangle} := mod\left(S3^{\langle 2\rangle}, 2\cdot\pi\right)$$

$$Y0 := \begin{pmatrix} 3.5 \\ 3 \end{pmatrix} \qquad S4 := rkfixe(Y0, 0, t_f, n, \boldsymbol{D}) \qquad S4^{\langle 2\rangle} := mod\left(S4^{\langle 2\rangle}, 2\cdot\pi\right)$$